CONNECTION THEORY OF CONICAL WORM GEAR DRIVES

S. BODZÁS¹, I. DUDÁS²

¹²College of Nyíregyháza, Department of Technical Preparatory and Production Engineering
H-4400 Nyíregyháza, Sóstói u. 9-11., HUNGARY
¹²University of Miskolc, Department of Production Engineering, H-3515 Miskolc, Egyetemváros, HUNGARY
E-mail: bodzassandor@nyf.hu
E-mail: illes.dudas@uni-miskolc.hu

We worked out a mathematical modell for the production geometry and mathematical analysis of spiroid worm gear drives. This modell is adapted for analysis of spiroid worm gear drives with random profile. Using this modell it could be possible for defining of the equations of cog surfaces, the surface normal vector, contact curves and connection surface in one concrete case.

Keywords: spiroid, transformation matrix, normal vector

Introduction

In technical practice conical worm surfaces, which can be used in many ways, are most widely applied as a function surface of conical worms. The conical worm – crown wheel pairs spiroid drive, can be used for example as jointless drives of robots and tool machines [1].

The jointless drives are attained by simply shifting (setting) the worm in an axial direction. The cog surface of the conical worm of the spiroid drives (Fig. 1) can be attained the same way as that of the cylindrical worm, but besides the axial shift of the hob, a tangential shift must be done depending on the conicity of the worm. Different – evolvent, Archimedean and convolute – helical surfaces can be defined in case of spiroid worm surface similar to the line surface cylindrical worm.

The dentation of crown wheel is produced with hob which tiler surface is similar to conical worm surface. This is called direct motion mapping.

With these modern drive pairs, which are characterized by favourable hidrodinamic conditions, great strength and high efficiency, the energy loss in the gear can be reduced significantly [1].

In power dissipation it is important to apply those cog geometrical characteristics which result in good connection terms.

Defining of the spatial coordination systems

![Figure 1: Spiroid worm gear drive](image1)

![Figure 2: Evading rotation axis coordinate system for defining of cog surfaces](image2)
We worked out this model based on the General mathematical model of Dr. Illés Dudás [1, 2]. Defining of minimum four coordinate systems are needed for analysing of motion transmission between evading axis and defining of cog surface describing spatial coordinates: two fixed rotation coordinate systems for the first part $K_{1F} (x_{1F}, y_{1F}, z_{1F})$ and the second part $K_{2F} (x_{2F}, y_{2F}, z_{2F})$ and two standing coordinate systems for the first part $K_1 (x_1, y_1, z_1)$ and the second part $K_2 (x_2, y_2, z_2)$, where the positions of the rotating coordinate systems can be defined (Fig. 2).

The rotation axis of the elements are $z_1$ and $z_2$, the turning direction is positive watching from the directions of the axis (opposite for the clock working), the turning angles, the motion parameters are $\phi_1$ and $\phi_2$.

The transformation matrixes between the rotation coordinate system $K_{1F} (x_{1F}, y_{1F}, z_{1F})$ for the first part and the standing coordinate system $K_1 (x_1, y_1, z_1)$ for the first part are:

$$M_{1,F,1} = \begin{bmatrix} \cos \phi_1 & -\sin \phi_1 & 0 & 0 \\ \sin \phi_1 & \cos \phi_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

The transformation matrixes between the standing coordinate system $K_1 (x_1, y_1, z_1)$ for the first part and the standing coordinate system $K_2 (x_2, y_2, z_2)$ for the second part are:

$$M_{1,F,2} = \begin{bmatrix} \cos \phi_1 & -\sin \phi_1 & 0 & 0 \\ \sin \phi_1 & \cos \phi_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2)$$

The transformation matrixes between the rotation coordinate system $K_{1F} (x_{1F}, y_{1F}, z_{1F})$ for the second part and the rotation coordinate system $K_{2F} (x_{2F}, y_{2F}, z_{2F})$ for the second part are:

$$M_{2,F,1} = \begin{bmatrix} \cos \phi_2 & \sin \phi_2 & 0 & 0 \\ -\sin \phi_2 & \cos \phi_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (5)$$

$$M_{2,F,2} = \begin{bmatrix} \cos \phi_2 & -\sin \phi_2 & 0 & 0 \\ \sin \phi_2 & \cos \phi_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (6)$$

The transformation matrixes between the rotation coordinate system $K_{1F} (x_{1F}, y_{1F}, z_{1F})$ for the first part and the rotation coordinate system $K_{2F} (x_{2F}, y_{2F}, z_{2F})$ for the first part are:

$$M_{2,1,F} = M_{2,F,2} \cdot M_{2,1} \cdot M_{1,F,1} = \begin{bmatrix} -\cos \phi_1 \cdot \cos \phi_2 & \sin \phi_1 \cdot \sin \phi_2 & \sin \phi_2 & 0 \\ -\sin \phi_1 \cdot \cos \phi_2 & \cos \phi_1 \cdot \sin \phi_2 & -\cos \phi_2 & 0 \\ \sin \phi_2 & \cos \phi_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (7)$$

$$M_{2,1,F} = M_{2,F,2} \cdot M_{2,1} \cdot M_{1,F,2} = \begin{bmatrix} -\cos \phi_1 \cdot \cos \phi_2 & \sin \phi_1 \cdot \sin \phi_2 & \sin \phi_2 & 0 \\ -\sin \phi_1 \cdot \cos \phi_2 & \cos \phi_1 \cdot \sin \phi_2 & -\cos \phi_2 & 0 \\ \sin \phi_2 & \cos \phi_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (8)$$

The equations of conical thread surface

The $r_g$ leading curve is given in the $K_0 (x_0, y_0, z_0)$ tool coordinate system and its equation of the $\eta$ coordinate function. That is:

$$r_g = r_g (\eta) \quad (9)$$

Since we consider the $\eta$ coordinate an independent variable, the equation of the leading curve is:

$$r_g = \xi(\eta) \cdot i + \eta \cdot j + \zeta(\eta) \cdot k \quad (10)$$

Carrying out a $p_z$ axial and $p_r$ radial helical motion of the $K_0 (x_0, y_0, z_0)$ coordinate system – which includes the $r_g$ leading curve – along the $z$ axis and the $y$ axis alternatively includes, the leading curve touches a conical helical surface in the $K_{1F} (x_{1F}, y_{1F}, z_{1F})$ an independent position and equals $K_0$ coordinate system before the helical motion (Fig. 4).
The helical surface touched by \( \vec{r}_\varphi \) curve in the \( K_{1F} (x_{1F}, y_{1F}, z_{1F}) \) coordinate system is:

\[
\vec{r}_{1F} = M_{1F,0} \cdot \vec{r}_\varphi
\]

The transformation matrix between the two coordinate systems is:

\[
M_{1F,0} = \begin{bmatrix}
\cos \vartheta & -\sin \vartheta & 0 & 0 \\
\sin \vartheta & \cos \vartheta & 0 & p_r \cdot \vartheta \\
0 & 0 & 1 & p_a \cdot \vartheta \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

That is:

\[
x_{1F} = \xi(\eta) \cdot \cos \vartheta - \eta \cdot \sin \vartheta \\
y_{1F} = \xi(\eta) \cdot \sin \vartheta + \eta \cdot \cos \vartheta + p_r \cdot \vartheta \\
z_{1F} = \zeta(\eta) + p_a \cdot \vartheta
\]

We gave the equations of the helical surface in the \( K_{1F} (x_{1F}, y_{1F}, z_{1F}) \) rotation coordinate system (13).

The given describing thread surface \( \vec{r}_\varphi = r_{\varphi}(\eta, \vartheta) \) two parameters vector – scalar function can be transformed from the \( K_{1F} \) coordinate system to the \( K_{2F} \) coordinate system:

\[
\vec{r}_{2F} = M_{2F,1F} \cdot \vec{r}_{1F}
\]

Subject to:

\[
\varphi_2 = i_{2F} \cdot \varphi_1
\]

**Direct case**

The \( \vec{r}_\varphi = r_{\varphi}(\eta, \vartheta) \) is given, the two parametric vector–scalar function can be transformed from the \( K_{1F} \) coordinate system to the \( K_{2F} \) coordinate system:

\[
\vec{r}_{2F} = M_{2F,1F} \cdot \vec{r}_{1F}
\]

The \( \eta \) and \( \vartheta \) parameters are the curve line coordinates of the surface.

We suppose the surface is continuous on the working parts of cogs, it is continuous function of \( \eta \) and \( \vartheta \) parameters; two coordinate lines have to be crossed through on every \( M \) point of the surface: a) \( \eta = \text{const} \), b) \( \vartheta = \text{const} \) the tangents of this lines do not coincide in this point. The working parts of the cog surfaces could contain only general points.

The normal vector belong to the surface and the tangent plane will be decided only in the general point.

The plane defined by the tangents of the \( \frac{\partial r_{\varphi}}{\partial \eta} \) and \( \frac{\partial r_{\varphi}}{\partial \vartheta} \) parameter lines is the tangent plane of the surface in the given point. The surface normal vector \( \vec{n}_{\varphi} \) is perpendicular for the tangent plane and it can be defined:

\[
\vec{n}_{\varphi} = \frac{\partial r_{\varphi}}{\partial \eta} \times \frac{\partial r_{\varphi}}{\partial \vartheta}
\]

The normal vector in the \( K_{1F} \) coordinate system is:

\[
\vec{n}_{1F} = \frac{\partial r_{\varphi}}{\partial \eta} \times \frac{\partial r_{\varphi}}{\partial \vartheta} = \begin{bmatrix}
\vec{i} \\
\vec{j} \\
\vec{k}
\end{bmatrix}
\]

The relative velocity of the two surfaces can be determined in coordinate system \( K_{2F} \) using the transformation between \( K_{1F} (x_{1F}, y_{1F}, z_{1F}) \) coordinate system for worm and the \( K_{2F} (x_{2F}, y_{2F}, z_{2F}) \) coordinate system for worm gear:

\[
\psi_{2F}^{(12)} = \frac{d}{dt} \cdot r_{2F} = \frac{d}{dt} (M_{2F,1F}) \cdot r_{1F}
\]

The vector \( \psi_{2F}^{(12)} \) should be transformed into coordinate system \( K_{1F} (x_{1F}, y_{1F}, z_{1F}) \) to determine the necessary connection surface, so:

\[
\psi_{1F}^{(12)} = M_{1F,2F} \cdot \psi_{2F}^{(12)} = M_{1F,2F} \cdot \frac{d}{dt} (M_{2F,1F}) \cdot r_{1F} = P_1 \cdot r_{1F}
\]

where:

\[
P_1 = M_{1F,2F} \cdot \frac{d}{dt} (M_{2F,1F})
\]

the matrix for kinematic generation.
The application of the model

We designed a conical worm of which we carried out the virtual model and using the mathematical model (Fig. 6) we carried out the virtual model of the connecting worm gear (Fig. 7).

![Figure 6: Defining of the coordinate systems](image)

![Figure 7: Our designed worm gear drive model](image)

**Summary**

We worked out a mathematical model for production geometry and mathematical analysis of spiroid worm. This model is appropriate for every spiroid worm gear drives with random profile.

We designed a spiroid worm gear drive and using this model we carried out the virtual model of this drive pair.

**REFERENCES**