MODELING AND PARAMETER SENSITIVITY ANALYSIS OF A SYNCHRONOUS MOTOR

L. NEUKIRCHNER, A. FODOR, A. MAGYAR

University of Pannonia, Department of Electrical Engineering and Information Systems, Veszprém, HUNGARY
E-mail: foa@almos.uni-pannon.hu

A simple dynamic model of a synchronous motor is developed in this paper based on first engineering principles that describe the mechanical phenomena together with the electrical model. The constructed state space model consists of nonlinear state equations and bi-linear output equations. The developed model has been verified under the usual regulated operating conditions when the speed and the torque are controlled, the manipulated input is the network voltage and the exciter voltage. The effect of load on the controlled synchronous motor has been analyzed by simulation using PI controllers. Model parameter sensitivity analysis has been applied to determine the model parameters to be estimated.

Keywords: synchronous machine, dynamic state space model, sensitivity analysis, parameter estimation

Introduction

Classical synchronous motors are widely used machines when constant speed is necessary. The speed control of synchronous machines is a difficult problem since the motor speed is a linear function of the network frequency. For the control of the synchronous motor (SM) we have to use an inverter which generates the three phase sinusoidal electrical network and a DC power supply which provides the exciter voltage.

The final aim of our study is to design a controller that regulates the speed and the torque of the synchronous motor. Because of the specialties and great practical importance of synchronous machines in industry, their modeling for control purposes is well investigated in the literature. Besides of the basic textbooks (see e.g. [1, 2]), there are papers that describe the modelling and use the developed models for the design of various controllers.

The aim of this paper is to perform the parameter sensitivity analysis of a simple dynamic model of a synchronous motor. The result of this analysis will be the next step of the parameter estimation.

The model of the synchronous motor

Modelling assumptions

For constructing the synchronous motor model, let us make the following assumptions:

- a symmetrical tri-phase stator winding system is assumed,
- one field winding is considered to be in the machine,
- all the windings are magnetically coupled,
- the flux linkage of the windings is a function of rotor position,
- the copper losses and the slots in the machine are neglected,
- the spatial distribution of stator fluxes and apertures wave are considered to be sinusoidal,
- the stator and rotor permeability are assumed to be infinite.
- It is also assumed that all the losses due to wiring, saturation and slots can be neglected.

The four windings (three stators and one rotor) are magnetically coupled. Since the magnetic coupling between the windings is a function of the rotor position, the flux linkage of the windings is also a function of the rotor position. The actual terminal voltage \( v \) of the windings can be written in the form

\[
v = \pm \sum_{j=1}^{J}(r_j \cdot i_j) \pm \sum_{j=1}^{J}(\lambda_j)
\]

where \( i_j \) are the currents, \( r_j \) are the winding resistances, and \( \lambda_j \) are the flux linkages.

The positive directions of the stator currents point in the synchronous motor terminals.

Thereafter, the two stator electromagnetic fields, both travelling at rotor speed, were identified by decomposing each stator phase current under steady state into two components, one in phase with the electromagnetic field and another phase shifted by 90°. With the above, one can construct an air-gap field with its maximal aligned to the rotor poles (d axis), while the other is aligned to the \( q \) axis (between poles). This method is called the Park’s transformation [3, 4].
As a result, the vector voltage equation is:
\[ v_{d}i_{d} + v_{q}i_{q} = R_{ab}i_{ab} + R_{bc}i_{bc} + R_{ca}i_{ca} + L_{ab}\frac{d}{dt}i_{ab} + L_{bc}\frac{d}{dt}i_{bc} + L_{ca}\frac{d}{dt}i_{ca} \]
where \( v_{d} \) and \( v_{q} \) are the direct and the quadratic components of the stator voltage of the synchronous motor, \( i_{d} \) and \( i_{q} \) are the direct and the quadratic components of the stator current, while \( v_{F} \) and \( i_{F} \) are the exciter voltage and current.

In matrix form, the voltage equation is:
\[ L_{es}i_{es} + L_{rs}i_{rs} = v_{es} + v_{rs} \]
where \( v_{es} \) is the neutral voltage and \( R_{F} = r_{F} \).

Using Park’s transformation is replaced with the equations for the d-q voltage components:
\[ v_{d} = L_{d}\frac{d}{dt}i_{d} + kM_{F}i_{q} \]
\[ v_{q} = L_{q}\frac{d}{dt}i_{q} - kM_{F}i_{d} \]
where \( \omega = \omega_{d} \).

The state space model for the currents is obtained by expressing \( i_{es} \) from (11), i.e.
\[ i_{es} = L_{d}^{-1} \cdot R_{es} \cdot i_{es} - L_{q}^{-1} \cdot v_{es} \]

Power and torque equations

The electrical energy of the SM is a sum of the following mechanical equations.
\[ dW_{Electr} = dW_{Mech} + dW_{Field} + dW_{\Omega} \]
Time derivative of the energy equation is the power equation, which represents the energy change:
\[ P_{Mech} = P_{Electr} - P_{Field} - P_{\Omega} \]
Torque is obtained from dividing power by angular velocity.

\[ T_{\text{Mech}} = \lambda_s i_q - \lambda_q i_d \quad (13) \]

\[ \omega = \frac{d\theta}{dt} \quad (14) \]

The torque from field energy is given by the torque equation:

\[ T_{\text{Field}} = (i_0 \frac{\lambda_d}{d\theta} + i_d \frac{\lambda_{dd}}{d\theta} + i_q \frac{\lambda_{dq}}{d\theta}) \quad (15) \]

Afterwards the torque is:

\[ T_{\text{Mech}} = T_{\text{Electr}} - T_{\text{Field}} - T_{\text{Dump}} \quad (16) \]

From Newton’s law of motion we can write the speed and torque equation:

\[ 2H\dot{\omega} = T_{\text{electr}} - T_{\text{mech}} - D\omega \quad (17) \]

where D is a damping constant. We can write the input power in the following equation:

\[ P_{\text{elec}} = \omega(\lambda_s i_q - \lambda_q i_d) - \left( i_0 \frac{\lambda_d}{d\theta} + i_d \frac{\lambda_{dd}}{d\theta} + i_q \frac{\lambda_{dq}}{d\theta} \right) \quad (18) \]

\[ -r(i_0^2 + i_d^2 + i_q^2) - 3r_i^2 i_0^2 \]

(In balanced condition \( i_0 = 0 \).) After we can compute the accelerating torque:

\[ T_{\text{Acc}} = \frac{T_{\text{Electr}}}{3} - T_{\text{Mech}} - T_{\text{Dump}} = \frac{T_{\text{Electr}} - T_{\text{Mech}} - T_{\text{Dump}}}{3} \]

To compute electrical torque we should write the direct and quadratic part of the stator flux from equation (12):

\[ \lambda_d = L_d i_d + k M_F i_F \]
\[ \lambda_q = L_q i_q \quad (19) \]

After it we write electrical torque can be expressed:

\[ T_{\text{elec}} = \begin{bmatrix} L_d i_q \\ k M_F i_q - L_q i_d \\ i_F \\ i_q \end{bmatrix} \]

\[ = \frac{T_{\text{Mech}}}{\tau_j} \quad (20) \]

Using \( \dot{\omega} = \frac{T_{\text{Acc}}}{dt} \) it is possible compute the speed of the synchronous machine. The motion equation is as follows:

\[ \dot{\omega} = \left[ \frac{L_d i_q}{3\tau_j} \quad k M_F i_q - \frac{L_q i_d}{3\tau_j} \quad -\frac{D}{\tau_j} \right] \cdot \begin{bmatrix} i_d \\ i_F \\ i_q \end{bmatrix} + \frac{T_{\text{Mech}}}{\tau_j} \quad (21) \]

The loading angle (\( \delta \)) of the synchronous motor is

\[ \delta = \delta_0 + \int_0^\omega (\omega - \omega_r) dt \quad (22) \]

that can be differentiated to obtain the time derivative of \( \delta \)

\[ \dot{\delta} = \omega - \omega_r \quad (23) \]

Altogether, there are five state variables: \( i_d \), \( i_F \), \( i_q \), \( \omega \) and \( \delta \). The input variables (i.e. manipulable inputs and disturbances) are \( T_{\text{mech}} \), \( v_F \), \( v_d \) and \( v_q \). Observe that the state equations are bilinear in the state variables.

The outputs of the model are the speed (\( \omega \)) of the motor and the loading angle (\( \delta \)) of the SM.

### Model analysis

The state space model (10, 21, 23) has been verified by simulation against engineering intuition using parameter values of a similar machine [5]. After the basic dynamical analysis, eleven parameters have been selected for sensitivity analysis.

#### Motor parameters

The parameters are described only for phase a since the machine is assumed to have symmetrical tri-phase stator windings system. The stator mutual inductances for phase a are:

\[ L_{ab} = L_{ba} = -M_s - L_{m0}(2(\Theta - \frac{\pi}{6})) \]
\[ L_{bc} = L_{cb} = -M_s - L_{m0}(2(\Theta - \frac{\pi}{2})) \quad (24) \]
\[ L_{ca} = L_{ac} = -M_s - L_{m0}(2(\Theta + \frac{5\pi}{6})) \]

where \( M_s \) is a given constant. The phase a to rotor mutual inductances are given by (from phase windings to the field windings):

\[ L_{af} = L_{fa} = M_F \cos(\Theta) \]
\[ L_{bf} = L_{fb} = M_F \cos(\Theta - \frac{2\pi}{3}) \]
\[ L_{cf} = L_{fc} = M_F \cos(\Theta + \frac{2\pi}{3}) \quad (25) \]

where \( M_F \) is a given constant. Parameters \( L_d, L_o, L_0 \) and \( M_F \) used by the state space model (10, 21, 23) are defined as:

\[ L_d = L_s + M_s + \frac{3}{2} L_m \]
\[ L_q = L_s + M_s - \frac{3}{2} L_m \]
\[ L_0 = L_s - 2M_s \quad (26) \]
\[ M_F = \frac{L_{af}}{k} \]
\[ k = \sqrt{\frac{2}{3}} \]
Using the initial assumption of symmetrical tri-phase stator windings (i.e. \( r_a = r_b = r_c = r \)) the resistance of stator windings of the machine we denoted by \( r \). Resistance of the rotor exciter is represented by \( r_F \). Parameters \( L_d \) and \( L_q \) are the direct and quadratic stator inductances, \( L_F \) is the stator exciter inductance. \( D_{const} \) presents the damping constant, \( P \) is the proportional, \( I \) is the integrator parameter of the torque PI controller. The parameter values were obtained from the literature [1]:

\[
\begin{align*}
L_d & = 0.176 \, \text{H} & L_{AF} & = 0.1302 \, \text{H} \\
L_q & = 0.171 \, \text{H} & r & = 0.5577 \, \Omega \\
L_F & = 0.138 \, \text{H} & r_F & = 0.0195 \, \Omega \\
l_d & = 0.0126 \, \text{H} & D & = 2.004 \\
l_q & = 0.0126 \, \text{H} & I & = 0.1 \\
l_F & = 0.00849 \, \text{H} & P & = 0.05 \\
\end{align*}
\]

Eleven parameters of the synchronous motor have been selected for sensitivity analysis, and the sensitivity of the state variables has been investigated by Matlab dynamical simulation.

The equilibrium point of the state space model can be obtained from the steady-state version of state equations (10, 21, 23) using the above parameter values. The equilibrium point of the system is:

\[
\begin{align*}
\omega & = 1.779712 \\
i_f & = 2.978999 \\
i_d & = -2.046594 \\
i_q & = 0.375484 \\
P_m & = 2.912249 \\
\end{align*}
\]

The state matrix of the state space model (28) has the following numerical value in this equilibrium:

\[
\begin{bmatrix}
-3.3714 \cdot 10^{-2} & 8.8667 \cdot 10^{-4} & -5.8023 & -1.2242 \\
2.5277 \cdot 10^{-2} & -2.4983 \cdot 10^{-3} & 4.3503 & 9.1782 \cdot 10^{-1} \\
6.7901 \cdot 10^{-1} & 1.2970 \cdot 10^{-3} & 1.1685 & 2.4654 \cdot 10^{-1} \\
-6.2364 & -4.6031 & 3.5201 \cdot 10^{-2} & -5.3330 \cdot 10^{-1}
\end{bmatrix}
\]

The eigenvalues of the state matrix are:

\[
\begin{align*}
\lambda_{1,2} & = -3.399932 \cdot 10^{-2} \pm j4.068256 \\
\lambda_3 & = -4.082050 \cdot 10^{-3} \\
\lambda_4 & = -1.3064115 \cdot 10^{-3}
\end{align*}
\]

The real parts of the eigenvalues are negative but their magnitudes are small, thus the system is on the boundary of the stability domain.

**Stability analysis**

**Parameter sensitivity analysis**

The applied control method of the synchronous machine is a classical PI controller (Fig. 2) that ensures stability of the equilibrium point under small perturbations. The controlled output is the mechanical torque, the manipulated
input is the voltage. The proportional parameter of the PI controller of the torque is 0.05 and the integrator time is 0.1 in per units.

Model validation

The dynamical properties of the motor have been investigated. The response of the torque controlled motor has been tested under step-like changes of the exciter voltage. The simulation results are shown in Figs. 3, 4, 5 and 6 where the quadratic linkage inductance $l_q$, the damping constant $D$, the stator exciter resistance $r_F$ and the stator resistance $r$ are shown.

Sensitivity analysis

The aim of this section is to define parameter groups according to the system’s sensitivity on them. Linkage inductances $l_d$, $l_q$, $l_F$ are not used by the current model, only by the flux model. As it was expected, the model is insensitive for these parameters. Note, that the linkage inductance parameters are only used for calculating the fluxes of the machine (Fig. 3). Response to the exciter voltage step change of the controlled motor (means the deviation form the steady-state value) Sensitivity of the model to the controller parameters $P$ (proportional) and $I$ (integrator) and the damping constant $D$ has also been investigated.

This is why the output and the steady state value of the system variables do not change even for a considerably large change of $D$ (Fig. 4).

Figure 3: Model responses for the +90% changing of parameter $l_q$

Figure 4: Model responses for the +90% changing of parameter $D$

Figure 5: Model responses for the +90% changing of parameter $r$ stator resistance

Sensitivity analysis

Not sensitive: These are the linkage inductances $l_d$, $l_q$, $l_F$ and damping constant $D$. The state space model is insensitive for them, the parameter values cannot be determined from measurement data using any parameter estimation method.

Sensitive: The stator resistance $r$, the proportional controller parameter $P$, the integrator controller parameter $I$ and the stator inductances $L_d$ and $L_q$.

Critically sensitive: The rotor exciter resistance $r_F$ and the rotor exciter inductance $L_F$. 
The final aim of is to develop a simple yet detailed state space model of the induction motor for control purposes which gives us the possibility to develop and analyze different control strategies for the synchronous motor.

**ACKNOWLEDGEMENT**

We acknowledge the financial support of this work for the Hungarian State and the European Union under the TAMOP-4.2.1/B-09/1/KONV-2010-0003 project.

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